This book consists mainly of a table of 7S values of the real and imaginary parts of  $P_{-1/2+i\tau}(ix)$ , in floating-point format, for  $\tau = 0(0.01)15$  and x = 0(0.1)2(0.2)5(0.5) 10(10)60.

As noted in the preface, the present table constitutes a natural sequel to a series of four earlier Russian tables [1]–[4] of the same function, wherein the tabular arguments were limited to real numbers.

A valuable introduction includes a set of formulas permitting the extension of the tables to corresponding negative values of  $\tau$  and x. Also included therein are asymptotic series for Re  $P_{-1/2+i\tau}$  and Im  $P_{-1/2+i\tau}$ , as well as expressions for these functions in terms of hypergeometric functions.

The introductory text contains a description of the tables and a brief discussion of their construction. Interpolation with respect to  $\tau$  is shown to be feasible to full tabular precision by means of Lagrange formulas for three and four points. On the other hand, it is noted that such interpolation with respect to x is not practical in these tables, and direct calculation by the appropriate formula in the introduction is recommended by the author.

As noted in the introduction, these functions are encountered in applications of the Mehler-Fock transformation; in particular, they are useful in the solution of certain mixed boundary value problems in mathematical physics, such as the distribution of electricity on hyperboloids of revolution. Reference to such applications is included in the appended bibliography of nine titles.

J. W. W.

1. M. I. ŻHURINA & L. N. KARMAZINA, Tablifsy funktsii Lezhandra  $P_{-1/2+17}(x)$ , Tom I, Akad. Nauk SSSR, Moscow, 1960. [See Math. Comp., v. 16, 1962, pp. 253-254, RMT 22.] 2. M. I. ŻHURINA & L. N. KARMAZINA, Tablifsy funktsii Lezhandra  $P_{-1/2+17}(x)$ , Tom II, Akad. Nauk SSSR, Moscow, 1962. [See Math. Comp., v. 18, 1964, pp. 521-522, RMT 79(a); ibid., v. 19, 1965, p. 692, RMT 123, for a brief review of English translations of volumes 1 and 2.]

3. M. I. ŹHURINA & L. N. KARMAZINA, Tablitsy i formuly dlia sfericheskikh funktsii  $P^{m}_{-1/2+17}(z)$ , Akad. Nauk SSSR, Moscow, 1962. [See Math. Comp., v. 18, 1964, pp. 521–522, RMT 79(b); ibjd., v. 21, 1967, pp. 508–509, RMT 66, for a review of an English translation.] 4. M. I. ŹHURINA & L. N. KARMAZINA, Tablitsy funktsii Lezhandra, Akad. Nauk SSSR, Moscow, 1963.

49 [7].—HENRY E. FETTIS, JAMES C. CASLIN & KENNETH R. CRAMER, An Improved Tabulation of the Plasma Dispersion Function and Its First Derivative—Part I— Argument with Positive Imaginary Part; Part II—Argument with Negative Imaginary Part: Zeros and Saddle Points, Reports ARL 72-0056 and 72-0057, Aerospace Research Laboratories, Air Force Systems Command, United States Air Force, Wright-Patterson Air Force Base, Ohio, July 1972, Part I, IV + 408 pp., 28 cm. and Part II, IV + 434 pp., 28 cm. Copies obtainable from the National Technical Information Service, Operations Division, Springfield, Virginia 22151.

Let

(1) 
$$z(\rho) = \pi^{-1/2} \int_{-\infty}^{\infty} \frac{e^{-t^2} dt}{t-\rho}, \quad \rho = x + iy, y > 0.$$

Then

(2) 
$$z(\rho) = 2ie^{-\rho^2} \int_{-i\rho}^{\infty} e^{-t^2} dt = 2ie^{-\rho^2} \operatorname{Erfc} (-i\rho),$$

with no restrictions on  $\rho$ . We also have

(3) 
$$z(\rho) = i\pi^{1/2}\omega(\rho)$$

where  $\omega(\rho)$  is the function considered by Fried and Conte [1], and Faddeeva and Terent'ev [2]. Let  $\mu = t - x$  in (1). Then

(4) 
$$z(\rho) = z(x, y) = \pi^{-1/2} \int_0^\infty \frac{u e^{-u^2} \sinh 2ux \, du}{u^2 + y^2} + iy \pi^{-1/2} \int_0^\infty \frac{e^{-u^2} \cosh 2ux \, du}{u^2 + y^2}$$

and these integrals are essentially the so-called Voigt functions. Both of the latter integrals are easily evaluated using the trapezoidal rule. For details on this technique, see Hunter and Regan [3], and the references given therein. Using this procedure, the present authors tabulate in Part I,  $z(\rho)$  and  $z'(\rho) = -2 - 2\rho z(\rho)$  for x = 0(0.1) 20, y = 0(0.1)10, 11S. The Part II tables are as above, except that -y = 0(0.1)10. Part II also contains the first 200 zeros of  $z(\rho)$  and  $z'(\rho)$ , 11S, and the first 200 zeros of Erf  $(\rho)$ , 11S. Each part has an errata insert which pertains only to the introduction and has no bearing on the numerical data. In the tables, the first and second columns listed, for example, under  $z(x, y) \equiv z(\rho)$  are the real and imaginary parts of  $z(\rho)$ , respectively. The authors remark that the Fried and Conte [1] tables were found to contain inaccuracies, particularly when y < 0, and that the present work was done to fill the need for a more accurate tabulation. Certainly, this is the most extensive tabulation of the error function available.

## Y. L. L.

1. B. D. FRIED & S. D. CONTE, The Plasma Dispersion Function: The Hilbert Transform of the Gaussian, Academic Press, New York, 1961. (See Math. Comp., v. 17, 1963, pp. 94–95.) 2. V. N. FADDEVA & N. M. TERENT'EV, Tables of Values of the Function  $\omega(z) = e^{-z^2} (1 + 2i\pi^{-1/2} \int_0^z e^{t^2} dt)$ , for Complex Argument, Pergamon Press, New York, 1961. (See Math. Comp., v. 16, 1962, p. 384.) 3. D. B. HUNTER & T. REGAN, "A note on the evaluation of the complementary error function," Math. Comp., v. 26, 1972, pp. 539–542.

50 [7].—HENRY E. FETTIS & JAMES C. CASLIN, A Table of the Inverse Sine-Amplitude Function in the Complex Domain, Report ARL 72-0050, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, April 1972, iv + 174 pp., 28 cm. Copies available from the Defense Documentation Center, Cameron Station, Alexandria, Virginia 22151.

The Jacobian elliptic functions with complex argument arise in numerous applications, e.g., conformal mapping and tabular values are available in [1] and [2]. Often, one desires the inverse function. This could be accomplished by inverse interpolation in the above tables. However, such a procedure is inconvenient and of doubtful accuracy, especially in some regions where a small change in the variable produces a large change in the function. Charts are available in [1] from which qualitatively correct values of the inverse could be deduced, but no prior explicit tabulation is known.

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